Optical-Model Analysis of "Quasielastic" (p,n) Reactions*

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Measured differential cross sections for (p,n) transitions between isobaric analog states are compared with the predictions of an optical model which includes an isobaric spin-dependent potential proportional to $\mathbf{t}\cdot\mathbf{T}_0$, where $\mathbf{t}$ and $\mathbf{T}_0$ are the isobaric spins of projectile and target, respectively. The magnitudes of the measured cross sections indicate a strength for this potential which is close to the symmetry potential found from analysis of elastic proton scattering. The shapes of the angular distributions give strong evidence for the radial shape of this potential to be peaked at the nuclear surface. The calculations are made in the distorted-wave Born approximation using optical potential parameters determined by fits to elastic scattering. Numerical studies are made of the sensitivity of the predictions to various parameter changes to determine the significance of the fits obtained to experiment. Finally, some discussion is given of the "quasi-inelastic" transitions to excited isobaric states, in terms of the collective-model description of inelastic scattering.

I. INTRODUCTION

EXPERIMENTAL studies[1] of the neutron spectra from (p,n) reactions have shown that this reaction excites quite strongly a state in the residual nucleus which is the isobaric analog[2] of the target ground state—that is, a state whose structure is the same as that of the target ground state (except for the replacement of a neutron by a proton) and which differs in energy essentially by just the Coulomb energy of the added proton. The two states have the same quantum numbers, in particular the same isobaric spin T_0, and differ only in the orientation of T_0 in isobaric-spin space. If the target has N neutrons and Z protons, then $T_{03}=\frac{1}{2}(N-Z)=T_0$ for the target, and $T_{03}=T_0-1$ for the residual nucleus. The (p,n) transition between these states then has many similarities to a super-allowed β decay. [Of course, this description implies that T_0 is a good quantum number even for heavy nuclei; this point has been discussed in detail elsewhere.[3] Further, if there is a high density of levels in the residual nucleus with the appropriate spin and parity, the analog state, and the (p,n) transition strength, may be spread over a narrow band of these levels.]

The great similarity of the initial and final states makes this (p,n) transition very much like an elastic scattering (hence the expression "quasielastic") in which the isobaric spin $\mathbf{t}$ of the projectile is "flipped." It also makes plausible an attempt to explain such transitions in terms of a simple one-body potential, by analogy with the optical-model potential for elastic scattering. Indeed, the optical-model potential can be easily generalized to include these transitions by allowing it to contain an isobaric-spin-dependent term.[2–4] The simplest such term is proportional to $\mathbf{t}\cdot\mathbf{T}_0$, the off-diagonal part of which, t_+T_{0-}, can induce a (p,n) transition between the $T_{03}=T_0$ and $T_{03}=T_0-1$ states. There is, already, evidence for such a term from optical-model analyses[5] of elastic proton scattering, where the strength of the potential required shows a linear dependence on the symmetry parameter $(N-Z)/A$. This may be interpreted as the diagonal contribution of the $\mathbf{t}\cdot\mathbf{T}_0$ term. This implies that neutron scattering should require a similar term, but of opposite sign. Neutron potential strengths certainly differ from those for protons by the right order of magnitude,[6] but unfortunately neutron-scattering data are usually not precise enough to allow an unambiguous determination of the $(N-Z)$ dependence.[7] Similar evidence is obtained from the single-particle bound states of neutrons and protons, and may be related to the symmetry term in the semi-empirical mass formula.[8]

The purpose of the present paper is to compare the predictions of this simple optical model for the quasi-elastic (p,n) transitions with the experimental results reported in the preceding paper.[9] In particular we investigate whether the (p,n) reactions are consistent with the symmetry term needed for elastic proton scattering.[5] [A preliminary analysis[10] of some early data[1] on the differential cross section for $^{51}V(p,n)$ at

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[1] J. D. Anderson and C. Wong, Phys. Rev. Letters **7**, 250 (1961); **8**, 442 (1962); J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. **126**, 2170 (1962); **129**, 2718 (1963).

[2] J. B. French, Ned. Tijdschr. Natuurk. **27**, 185 (1961); Nucl. Phys. **26**, 161 (1961); J. B. French and M. H. Macfarlane, *ibid.* **26**, 168 (1961); Phys. Letters **2**, 255 (1962); A. M. Lane and J. M. Soper, Nucl. Phys. **37**, 506 (1962).

[3] A. M. Lane and J. M. Soper, Phys. Rev. Letters **7**, 420 (1961); Phys. Letters **1**, 28 (1962); *ibid.* **1**, 33 (1962); Nucl. Phys. **37**, 663 (1962).

[4] A. M. Lane, Phys. Rev. Letters **8**, 171 (1962); Nucl. Phys. **35**, 676 (1962).

[5] F. G. Perey, Phys. Rev. **131**, 745 (1963); J. Benveniste, A. C. Mitchell, B. Buck, and C. B. Fulmer, *ibid.* **133**, B323 (1964); J. B. Ball, C. B. Fulmer, and R. H. Bassel, *ibid.* **135**, B706 (1964).

[6] F. G. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962) and private communication; F. G. Perey, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963).

[7] But see P. E. Hodgson, Phys. Letters **3**, 352 (1963).

[8] P. C. Sood, Nucl. Phys. **37**, 624 (1962); A. E. S. Green and P. C. Sood, Phys. Rev. **111**, 1147 (1958); G. R. Satchler, *ibid.* **109**, 429 (1958).

[9] J. D. Anderson, C. Wong, J. W. McClure, and B. D. Walker, Phys. Rev. **136**, B118 (1964).

[10] R. M. Drisko, R. H. Bassel, and G. R. Satchler, Phys. Letters **2**, 318 (1962).

12 MeV was very encouraging in this respect.] In addition, an attempt is made to learn something of the radial shape[11] of the isobaric-spin potential by considering two extremes, a uniform ("volume") distribution proportional to the real part of the main optical potential, and a surface-peaked distribution proportional to the derivative of the main potential.

An alternate approach to (p,n) reactions between analog states is to use shell-model wave functions and to take as interaction a sum of two-body forces between the projectile and target nucleons.[12] Such a study is under way and will be reported on later. This approach is more fundamental than the phenomenological potential model just described, but suffers from a number of difficulties. The effective two-body interaction is rather poorly known, although it may be argued that application to processes like this is one way to study it. In addition, the nuclear wave functions may not be sufficiently well known, although considerable effort has been expended recently on attempts to improve simple shell-model wave functions.[13]

For our present purpose, it is sufficient to note that this shell-model point of view does suggest that the intensity of the (p,n) reaction, or the strength and radial shape of the symmetry potential, should be rather sensitive to the orbitals occupied by the excess neutrons. Then we should not be surprised to find these quantities varying more from nucleus to nucleus than the simple potential model would indicate, and perhaps showing shell-closure effects. On the other hand, a mitigating feature recently pointed out[14] is that the effective two-body interaction between nucleons in nuclear matter itself depends upon the difference between neutron and proton densities. This implies that all the nucleons in the nucleus contribute to the symmetry potential, which would then show a more smooth behavior from nucleus to nucleus.

Finally, we should remark that the optical model used in the calculations reported here assumes a local potential, even though we have every reason to believe that a substantial part of the energy dependence of such a potential is due to nonlocality.[6] This procedure is consistent in the sense that we are considering an extension to (p,n) reactions of the optical potentials conventionally used to study elastic scattering. However, it is as well to bear in mind this restriction. Nonlocality will enter in two ways. First, it is known that, for the same elastic scattering, the wave function for a nonlocal potential is reduced in magnitude in the nuclear interior compared to that for a local potential.[6] This will tend to increase the importance of the surface region for the (p,n) reaction. Secondly, the i-spin-dependent interaction potential may itself be nonlocal. While it is known that an equivalent local potential can always be found to give the same scattering, it should be remembered that the i-spin-flip scattering which gives rise to the (p,n) reaction occurs a considerable way off the energy shell. (Owing to the Coulomb energy difference, the energy loss is $\frac{1}{2}$ or more of the incident energy for the medium-weight nuclei discussed here.) We might thus expect to find differences between the equivalent local potential needed for the (p,n) reaction and the symmetry-dependent part of the elastic local potential, if the nonlocality were important. These questions are being explored in more detail.

II. THEORY

We assume that the optical-model potential may be written in the form[4]

$$U=U_0(r)+U_1(r)(\mathbf{t}\cdot\mathbf{T}_0)/A\,, \tag{1}$$

where $A=N+Z$ is the mass number of the target. In addition, we have the Coulomb potential. For a smooth charge distribution this may be written

$$(\tfrac{1}{2}-t_3)U_c(r)\,, \tag{2}$$

where $t_3=+\frac{1}{2}$ for a neutron and $t_3=-\frac{1}{2}$ for a proton, so that U_c is the Coulomb potential felt by a proton. The Coulomb potential has two effects. First, the proton and neutron channels have different energies (for a given total energy of the system); that is to say, the energies of the analog states are separated by an amount equal to the Coulomb energy, and the (p,n) transition between them has a nonzero Q value. Secondly, while the total isobaric spin $\mathbf{T}=\mathbf{T}_0+\mathbf{t}$ is a good quantum number for the potential of Eq. (1), the Coulomb interaction (2) couples states with different T. Using these two potentials in the Schrödinger equation then leads to a set of coupled equations for each partial wave.[4,15] However, if we can neglect the Coulomb potential (2), and if we work with the total spin T, we obtain two uncoupled scattering amplitudes $f_T(\theta)$, corresponding to the two values $T=T_0\pm\frac{1}{2}$, for which the optical potential (1) has the values

$$U=U_0(r)+\tfrac{1}{2}A^{-1}U_1(r)\times\begin{pmatrix}T_0\\-T_0-1\end{pmatrix}$$

if

$$T=\begin{pmatrix}T_0+\frac{1}{2}\\T_0-\frac{1}{2}\end{pmatrix}. \tag{3}$$

A state of definite T, however, corresponds to a mixture (weighted by the appropriate Clebsch-Gordan coefficients[16]) of states with $t_3=\pm\frac{1}{2}$, that is, of neutron and

[11] T. Terasawa and G. R. Satchler, Phys. Letters **7**, 265 (1963).
[12] S. D. Bloom, N. K. Glendenning, and S. A. Moszkowski, Phys. Rev. Letters **3**, 98 (1959).
[13] For application of these techniques to analog states see K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Letters **2**, 169 (1962); S. Fujii, Nucl. Phys. **52**, 144 (1964); S. Fallieros, contributed paper to Symposium on Nuclear Spectroscopy with Direct Reactions, Chicago, 1964, Argonne National Laboratory Report ANL-6848 (unpublished); W. T. Pinkston (to be published).
[14] K. A. Brueckner and J. Dabrowski, Phys. Rev. **134**, B722 (1964).
[15] P. E. Hodgson and J. R. Rook, Nucl. Phys. **37**, 632 (1962).
[16] D. M. Brink and G. R. Satchler, *Angular Momentum* (Oxford University Press, New York, 1962).

proton states. We can then project out the parts of the $f_T(\theta)$ which correspond to the (p,n) transition, and this yields for the (p,n) cross section[4,15]

$$\frac{d\sigma}{d\omega}=\frac{2T_0}{(2T_0+1)^2}|f_{T_0+1/2}(\theta)-f_{T_0-1/2}(\theta)|^2. \quad (4)$$

Neglecting the Coulomb potential, however, corresponds to neglecting both the Coulomb scattering of the proton and the energy loss (or Q value) of the reaction. Except for the lightest nuclei, these are likely to be poor approximations. Already for ^{51}V the Q value is -8 MeV, and the Coulomb barrier is approximately 6 MeV, both of which are large fractions of the incident proton energies of 17 and 18 MeV with which we are concerned here. However, a more useful approach is possible when we recognize that the off-diagonal part of the potential in Eq. (1),

$$\Delta U\equiv\langle t_3=+\tfrac{1}{2},\ T_{03}=T_0-1|\,U\,|t_3=-\tfrac{1}{2},\ T_{03}=T_0\rangle$$
$$=-\tfrac{1}{2}A^{-1}U_1(r)(N-Z)^{1/2}, \quad (5)$$

is very small compared to the diagonal parts,

$$\langle t_3=\pm\tfrac{1}{2},\ T_{03}=T_0|\,U\,|t_3=\pm\tfrac{1}{2},\ T_{03}=T_0\rangle$$
$$=U_0(r)\pm\tfrac{1}{4}U_1(r)(N-Z)/A. \quad (6)$$

Optical-model analysis of proton scattering[5] indicates that U_0 is a well of approximately 50-MeV depth, while if U_1 is assumed to have the same shape as U_0, it has a strength of about 100 MeV. Then for ^{64}Zn, for example, the matrix element (5) has a value of less than 2 MeV, or about 3% of U_0. So we can treat ΔU, the off-diagonal part of U, in first-order perturbation theory; that is to say, in distorted-wave Born approximation.[17] This yields a transition amplitude

$$T_{pn}=\int d\mathbf{r}\ \chi_n^{(-)*}(\mathbf{k}_n,\mathbf{r})\Delta U(r)\chi_p^{(+)}(\mathbf{k}_p,\mathbf{r}) \quad (7)$$

for an incident proton with momentum $\mathbf{k}_p$ and emitted neutron with momentum $\mathbf{k}_n$. The corresponding differential cross section is

$$(d\sigma/d\omega)=(\mu/2\pi\hbar^2)^2(k_n/k_p)|\,T_{pn}|^2, \quad (8)$$

where μ is the reduced mass of the nucleon-target system. The $\chi(\mathbf{k},\mathbf{r})$ are the distorted waves which describe the elastic scattering of the proton before, and the neutron after, the charge exchange. This elastic scattering is due to the usual optical potentials (6), together with the Coulomb potential for the proton. In this way the distorted-wave theory is able to include the effects of the Coulomb scattering of the proton and the difference in the energies of the neutron and proton without any difficulty. As described below, the accuracy of the approximation has been verified by direct numerical comparison with the expression (4) in a case where the Coulomb potential is neglected (Sec. IV).

The details of the evaluation of the amplitude (7) have been given in many places.[17,18] In general, the optical potentials (6) contain some spin-orbit coupling; for the calculations reported here the form of potential adopted was

$$U(r)=-V(e^x+1)^{-1}+i4W_D(d/dx')(e^{x'}+1)^{-1}$$
$$+\mathbf{L}\cdot\boldsymbol{\sigma}V_{s0}(\hbar/m_\pi c)^2r^{-1}(d/dr)(e^x+1)^{-1}, \quad (9)$$

where

$$x=(r-r_0A^{1/3})/a,\quad x'=(r-r'_0A^{1/3})/a'.$$

The Coulomb potential from a uniform charge of radius $r_0A^{1/3}$ was included for the protons. The imaginary part of the potential (9) is peaked at the nuclear surface; a calculation is also reported below in which a volume absorption was used. The parameters were chosen to be those which give a good description of the observed elastic scattering of protons[5] and neutrons.[6] For self-consistency in using the model, the neutron and proton potentials should be related by Eq. (6). This condition can be satisfied approximately, but it is difficult to obey exactly because the neutron and proton energies are different and at least the parameter V is known to be energy-dependent. Further, the symmetry dependence of the imaginary potential W_D is quite unknown.[19] Fortunately, the results of the calculations are not overly sensitive to small changes in the parameters; this is explored numerically in Sec. IV.

It remains to specify the shape of the isobaric-spin-dependent term $U_1(r)$. Up to the present, calculations have been made assuming this has the same Woods-Saxon shape as the real part of the main potential $U_0(r)$. There is no compelling reason why this should be so, and some reasons for expecting a surface peak in U_1 were put forward recently.[11] In the present work, two extremes are used. For the first, which we call "volume," U_1 is taken to be proportional to $\mathrm{Re}U_0$, which corresponds to a symmetry-dependent real well depth V in Eq. (9),

$$V=V_0-V_1(\mathbf{t}\cdot\mathbf{T}_0)/A. \quad (10)$$

For the diagonal optical potentials (6) this becomes

$$V=V_0\pm\tfrac{1}{4}V_1(N-Z)/A, \quad (11)$$

with + for protons, − for neutrons, so that both V_0 and V_1 are positive numbers. Proton-scattering analyses have used this form,[5] and indicate that $V_1\approx100$ MeV. The second form used, which we call "surface," assumes

[17] N. Austern, in *Fast Neutron Physics, II*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963); and *Selected Topics in Nuclear Theory*, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963), and references contained therein.

[18] See, for example, R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962); G. R. Satchler, Nucl. Phys. **55**, 1 (1964). The calculations were done by the IBM-7090 using the code JULIE.

[19] The trend reported in Ref. 5 for W_D for protons to increase with $(N-Z)/A$ is almost certainly accidental. Recent work indicates the increase in W_D is associated with an over-all increase in the coupling to collective modes for the heavier nuclei. F. Perey (private communication).

that $U_1(r)$ is proportional to the radial derivative of $U_0(r)$, which is peaked at the surface $r=R=r_0A^{1/3}$. That is, we take

$$U_1(r)=-V_1'(d/dx)(e^x+1). \quad (12)$$

Since U_1 is small compared to U_0, this surface form is equivalent to a small symmetry-dependent increase in the radius R of U_0. This we may see from the Taylor expansion of the real part of U_0,

$$V_0f(r-R-\delta R)=V_0f(r-R)+\delta RV_0df(r-R)/dR\cdots. \quad (13)$$

Remembering that $dR=adx$, comparison with Eq. (11) shows

$$\delta R=a(V_1'/V_0)(\mathbf{t}\cdot\mathbf{T}_0)/A. \quad (14)$$

The proton-scattering analyses[5] may be used to yield an estimate of V_1'. For small changes, a change in well depth V and a change in radius R are approximately equivalent for elastic scattering, provided VR^n is kept constant (with $n\approx2$ at low and medium energies[20]). Comparison of Eqs. (10) and (14) then gives

$$V_1'\approx V_1R/na. \quad (15)$$

However, while it is difficult to distinguish between the volume and surface symmetry potentials by analysis of elastic-scattering data, the situation is quite different for (p,n) reactions between analog states. We shall see that the (p,n) angular distributions are quite different.

The two shapes for $U_1(r)$ chosen here are extreme, and the true shape may well be intermediate, partly volume with some surface peaking. Further, simplicity is the only reason for choosing the same values of radius R and diffuseness a in U_1 as in the real part of U_0. The presently available data are not sufficiently detailed to detect small differences in these parameters. In addition, we have assumed that $U_1(r)$ is real. The justification for this is less clear, since we have very little information on the symmetry dependence of ImU. Fortunately, an imaginary component in U_1 of the same relative size as in U_0 has rather little effect on the (p,n) reaction (see Sec. IV).

III. COMPARISON WITH EXPERIMENT

Potential Parameters

For the calculations reported here, except where otherwise stated, the parameters for the optical potential (9) were chosen to be those which reproduce the observed elastic scattering at these energies.[5,6] For both protons and neutrons we use the "geometrical" parameters $r_0=r_0'=r_c=1.25$ F, $a=0.65$ F, and $a'=0.47$ F. The real well depth for protons of energy E_p MeV was chosen to be[5]

$$V_p=53.3-0.55E_p+0.4(Z/A^{1/3})+27(N-Z)/A, \quad (16)$$

rounded to the nearest MeV. The strength for the symmetry dependence is approximately consistent with the values of V_1 deduced below from the (p,n) reactions; any inconsistency corresponds to a change in V_p of less than 1 MeV. The values of V_n for neutrons are less certain. Neutron-scattering data are well described by[6]

$$V_n=48-0.3E_n \quad (17)$$

for neutrons of energy E_n MeV. The elastic-scattering data do not demand a symmetry-dependent term; on the other hand, including such a term with the same strength as needed by proton data would not significantly change the agreement between experiment and theory. Further, for our present purpose a symmetry dependence of V_n would tend to be compensated for by a stronger energy dependence than indicated by Eq. (17). Both $N-Z$ and the Q value increase for the heavier nuclei, so any decrease in V_n due to symmetry dependence would be offset by an increase due to the lower energy of the neutrons. In fact, the V_n values implied by Eq. (16) for the nuclei considered here are very close to those given by Eq. (17). Calculations were first made with V_n given by Eq. (17), but later, as described below, variations in V_n were tried.

The imaginary potential strengths W_D are much less well determined. The values which fit proton scattering are roughly given by[5]

$$W_{Dp}\approx3A^{1/3}\pm1.5 \text{ MeV},$$

while fits to 14-MeV neutron scattering give[6]

$$W_{Dn}\approx9.6\pm1.1 \text{ MeV}.$$

Fortunately, as is discussed below, the (p,n) angular distributions do not change significantly with reasonable variations in W_D, while the magnitudes of the cross sections predicted are quite closely inversely proportional to both W_{Dn} and W_{Dp}. Since the predicted cross section is directly proportional to V_1^2, the value of V_1 we extract by comparison with experiment is proportional to the value of $(W_{Dn}W_{Dp})^{1/2}$ used in the distorted-waves calculation. The values chosen here were $W_{Dp}=12$ MeV and $W_{Dn}=9.6$ MeV.

Finally, the spin-orbit strengths were chosen equal for neutron and proton, $V_{so}=7$ MeV. In this way, all the parameters of the model are predetermined apart from the choice between "volume" and "surface" interaction and the strength V_1. Since in the distorted-wave approximation the angular distribution is independent of the value of V_1, but the magnitude of the

[20] A study was made of the $VR^n=$constant rule for the scattering of 17-MeV protons and 8-MeV neutrons from ^{56}Fe, by calculating the derivative of the cross section with respect to V and R. The value of n was found to be 2.20 (protons), 1.85 (neutrons) if the imaginary potential is kept fixed and only the real potential varied. [If the radius of the imaginary potential is varied with that of the real, the value of n becomes 2.65 (protons), 1.95 (neutrons).] Further, a re-analysis [F. Perey (private communication)] of 17-MeV proton-scattering data was made with V_p fixed at $44.85+(0.4Z/A^{1/3})$ MeV and the values of W_D and r_0 varied for optimum fits. The results are well correlated by the formula $r_0=1.236+0.26\ (N-Z)/A$. Using Eq. (15), and $n=2$, this corresponds to $V_1=82$ MeV. [Previously (Ref. 5) assuming a volume symmetry term, Eq. (10), a value $V_1=108$ MeV was obtained. Equating these values through Eq. (15) would imply $n=2.62$.]

TABLE I. Parameters (in MeV) for the optical-model predictions at 18.5 MeV shown in Figs. 1–3. The well depths V_p and V_n are for the "standard set" obtained from Eqs. (15) and (16). $V_1(V)$ is the strength for the volume form, Eq. (9), while $V_1(S)$ is the strength using the surface form, Eq. (11), obtained from Eq. (15) with $n=2$.

Element	$-Q$	V_p	V_n	$V_1(V)$	$V_1(S)$
^{45}Sc	7.6	48	45	90	90
^{48}Ti	7.8	48	45	81	96
^{51}V	8.1	48	45	87	111
^{52}Cr	8.3	48	45	125	100
^{55}Mn	8.6	48	45	90	100
^{56}Fe	9.0	48	45	73	82
^{59}Ni	9.5	47	46	90	83
^{59}Co	9.2	48	45	83	87
^{63}Cu	9.5	48	46	78	85
^{65}Cu	9.5	49	46	70	81
^{79}Se	10.6	50	46	88	118
^{88}Sr	11.3	50	46	99	131
^{89}Y	11.6	50	46	100	109
^{91}Zr	11.8	50	46	63	79
^{93}Nb	11.95	50	46	100	111

cross section is directly proportional to V_1^2, these two unknowns can be determined more or less independently.

The potential parameters described here are called the "standard set," and the values of V_p and V_n are listed in Table I.

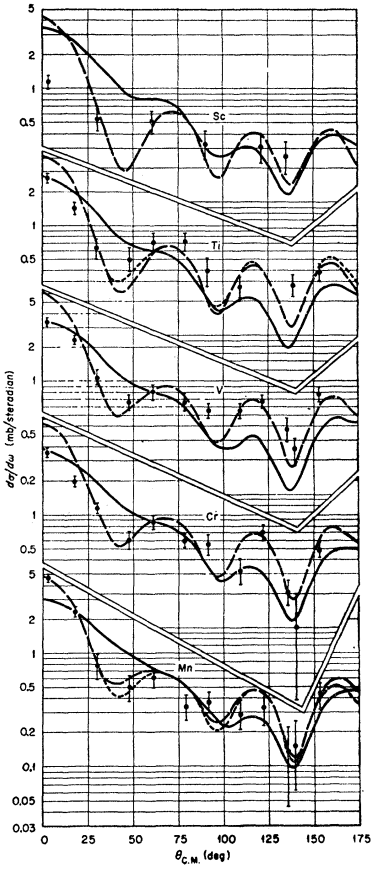

FIG. 1. Comparison of theory and experiment for quasielastic (p,n) reactions at 18.5 MeV. Full curves are for volume interaction, broken curves for surface interaction, with parameters given in Table I and the text. Dotted curve for Ti obtained using $V_n=48$ MeV, that for Mn using $V_n=43$ MeV.

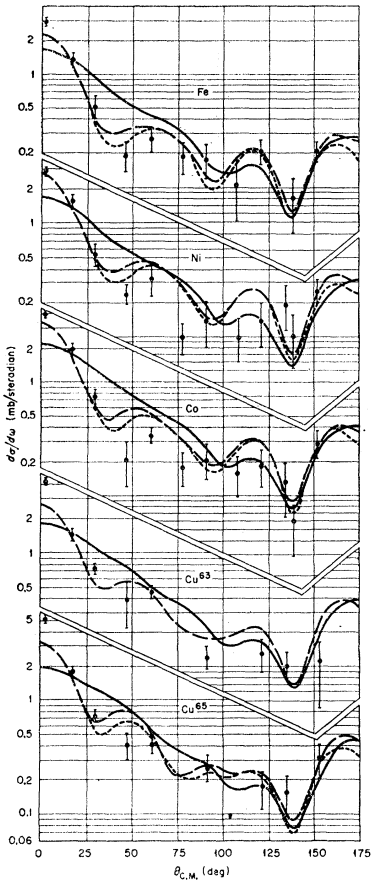

FIG. 2. Comparison of theory and experiment for quasielastic (p,n) reactions at 18.5 MeV. Full curves are for volume interaction, broken curves for surface interaction, with parameters given in Table I and the text. Dotted curve for Fe obtained using $V_n=44$ MeV, those for other targets using $V_n=43$ MeV.

Medium-Weight Nuclei

The characteristics of the experimental data have been discussed in the preceding experimental paper.[9] The data are compared in Figs. 1–4 with the optical-model predictions (calculated in distorted-wave Born approximation) using the "standard" parameters. Both the volume (full curves) and surface (broken curves) forms of the interaction are used. Further details are given in Table I; the strengths $V_1(S)$ given there for the surface interaction are obtained from Eq. (15) assuming $n=2$, so as to be directly comparable to the volume strengths $V_1(V)$. Experimentally, some targets were isotopic mixtures; the strengths V_1 in Table I were extracted assuming the isotope listed there. In fitting the data, less weight was given to the point at 3° because there is, perhaps, greatest experimental uncertainty associated with this angle. In any case, there is at least 10% uncertainty in the value of V_1 due to uncertainties in the choice of "best fit". In addition, there is a further 10% "error" in V_1 for Sc, Se, Sr, Y, and Zr due to uncertainties in the experimental absolute cross sections.[9]

The 18.5-MeV data for the nuclei Ti to Cu clearly provide evidence for the surface form for the interaction, chiefly because of the minimum which appears

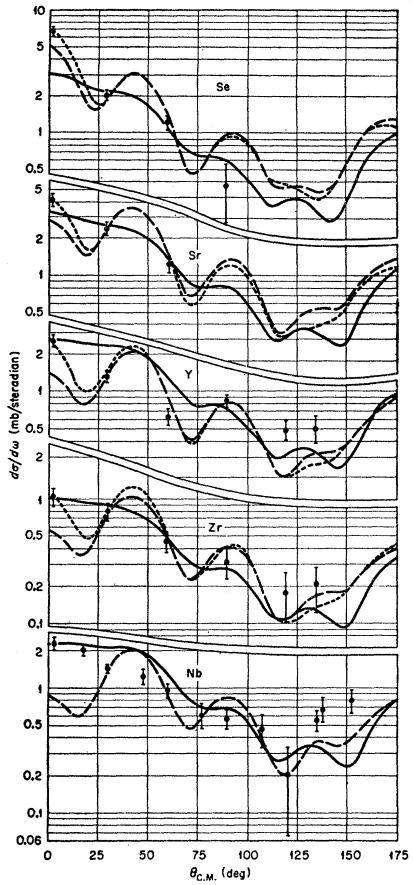

FIG. 3. Comparison of theory and experiment for quasielastic (p,n) reactions at 18.5 MeV. Full curves are for volume interaction, broken curves for surface interaction, with parameters given in Table I and the text. Dotted curves obtained using $V_n=43$ MeV, with the value of $V_1(S)$ changed to 131, 141, 120, and 90 MeV for Se, Sr, Y, and Zr, respectively.

around 30° to 40°, although the structure at wide angles also lends support to this. Except for Nb, the data for the other nuclei are not complete enough to choose between the two forms. Sc, Se, and Y seem to favor the surface form, while Sr and Zr are equally well fit by either. On the other hand, Nb prefers the volume form (indeed, a volume interaction with a radius some 10% smaller than the main optical potential radius gives a very good fit to the Nb data).

Similarly, the data at 17 MeV, shown in Fig. 4, do not unambiguously favor one or other form of the interaction. The earlier data[1] from V at 12 MeV are not very well reproduced by either form of the interaction, although here the volume shape disagrees less than the surface.

Nonetheless, it must be emphasized that the curves shown in these figures are predictions from the "standard set" of parameters and involve no parameter variations. As already discussed, there are some uncertainties in the parameters of the optical potentials; the effects of some of these are studied later, but the effects of small variations in V_n on the surface predictions for some nuclei are shown as dotted curves in Figs. 1 through 3. (These variations have very much less effect when the volume interaction is used.) The main change produced is in the first minimum of the angular distribution. There is perhaps an improvement in the fits to the heavier elements when V_n is reduced by 3 MeV, but the accuracy of the data does not allow any definite conclusions to be drawn.

The strengths $V_1(S)$ employed in Figs. 1 through 4 with the surface interaction are plotted against mass number A in Fig. 5. The errors (10%) shown on some points are those due to uncertainties in the experimental cross-section magnitudes[9]; it was thought best not to attempt to associate errors with the uncertainties in fitting the data, although these are at least 10%, and may be much more in some cases. In view of this, the values shown in Fig. 5 are probably all consistent with a single value, $V_1(S)=100\pm20$ MeV, although it is tempting to see there some evidence for shell structure.

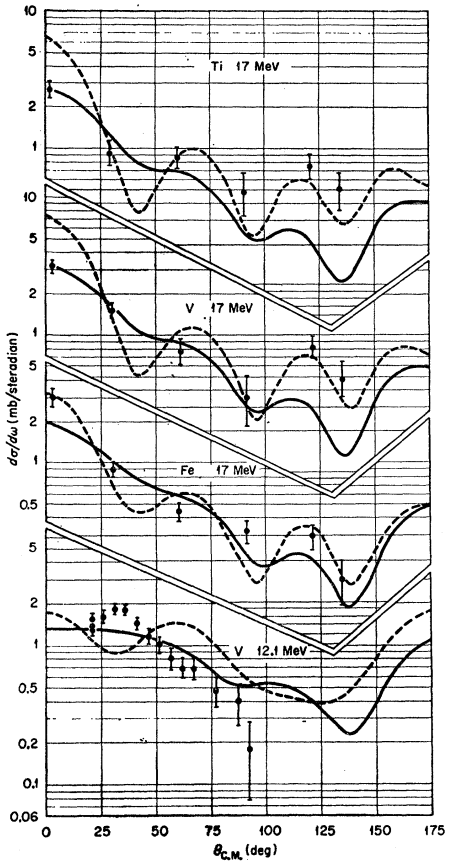

FIG. 4. Comparison of theory and experiment for quasielastic (p,n) reactions on Ti, V, and Fe at 17 MeV, and on V at 12.1 MeV. Full curves are for volume interaction, broken curves for surface interaction. The parameters are given in Table I and the text, except that $V_1(V)$ has the values 83, 87, 84, and 100 MeV, and $V_1(S)$ has the values 118, 118, 101, and 136 MeV for Ti, V(17), Fe, and V(12.1), respectively.

Further, it should be remembered that these values were adduced assuming fixed values of W_{Dp} (12 MeV) and W_{Dn} (9.6 MeV). However, these values are expected to vary somewhat from nucleus to nucleus, and, since the value of V_1 required is roughly proportional to the value of $(W_{Dp}W_{Dn})^{1/2}$ used, some of the fluctuations in V_1 may be due to this.

Despite these uncertainties, the over-all agreement between the data and the predictions of the simple potential model must be regarded as remarkably good, both in angular distribution and in magnitude. The lines drawn in Fig. 5 correspond to the values of V_1 obtained from fitting proton elastic scattering with a volume[5] or surface[20] symmetry potential.

Another feature worth noting is the absence of any marked difference in the data for even and odd targets. If the isobaric analog-state interpretation is correct, even targets allow only monopole, $l=0$, transitions, these being 0^+ to 0^+. Odd targets with spins $I>\frac{1}{2}$ also allow even moments $l=2, 4, \cdots 2I$. The optical-model description, however, only allows $l=0$ in these cases also, since the potential is assumed to be spherically symmetrical. The cross sections for even and odd targets are then predicted to be the same. We return to this point in Sec. V.

Light Nuclei

Although we have less anticipation of success with a simple potential model when it is applied to very light nuclei, these results are included for completeness. There is considerably greater uncertainty over the values of optical-model parameters to be used for light nuclei. However, on the basis of analyses of elastic scattering of neutrons and protons from these nuclei, it was decided to take $V_p=50$ MeV, $W_{Dp}=6.5$ MeV, $V_n=45$ MeV, and $W_{Dn}=6.5$ MeV as representative, the other parameters being the same as in the "standard set." A comparison of the predictions using these with the data for ground-state transitions is shown in Fig. 6. These are transitions between true mirror states. The curves for B, C, and N are normalized with $V_1=100$ MeV, while for Be we use $V_1=65$ MeV. The agreement with the data is surprisingly good. Except for N, which favors the surface interaction, there is no strong preference for one or other form of interaction. We note, however, that the surface interaction reproduces qualitatively the trend of the forward cross section as we go from N (minimum) to Be (maximum).

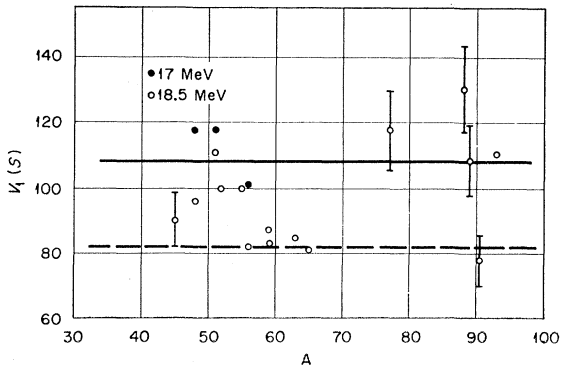

FIG. 5. The values of $V_1(S)$ used in Figs. 1 through 4 plotted against mass A. The straight lines are values obtained from proton-scattering analyses; the full line using a volume potential (Ref. 5), the broken line using a surface potential (Ref. 20).

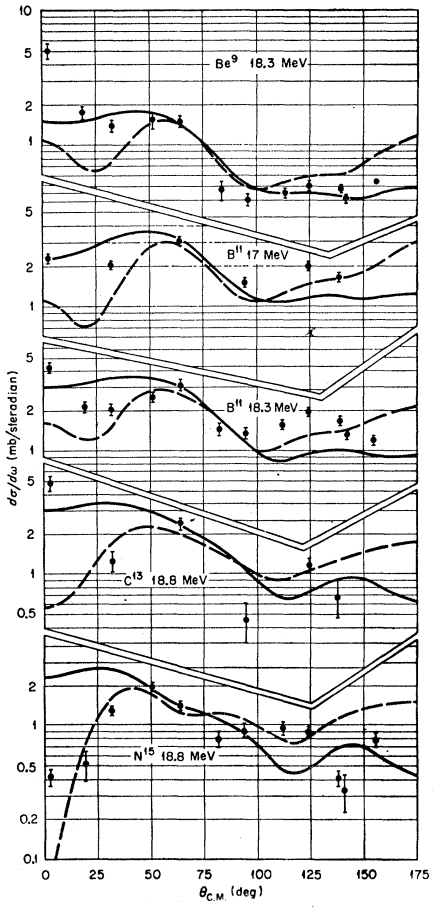

FIG. 6. Comparison of theory and experiment for (p,n) reactions between mirror states in light nuclei. Full curves calculated with volume interaction, broken curves with surface interaction, and $V_1=100$ MeV except for ^{9}Be, where $V_1=65$ MeV.

The *details* of the angular distributions for these light nuclei are quite sensitive to the choice of optical-model parameters. However, it was not deemed appropriate at the present time to make an extensive study of such variations. Rather we content ourselves with remarking that the simple optical-model picture gives a good over-all account of these transitions.

IV. MODEL STUDIES

Approximation Methods

We first established that the distorted-wave Born approximation was valid by comparing its predictions

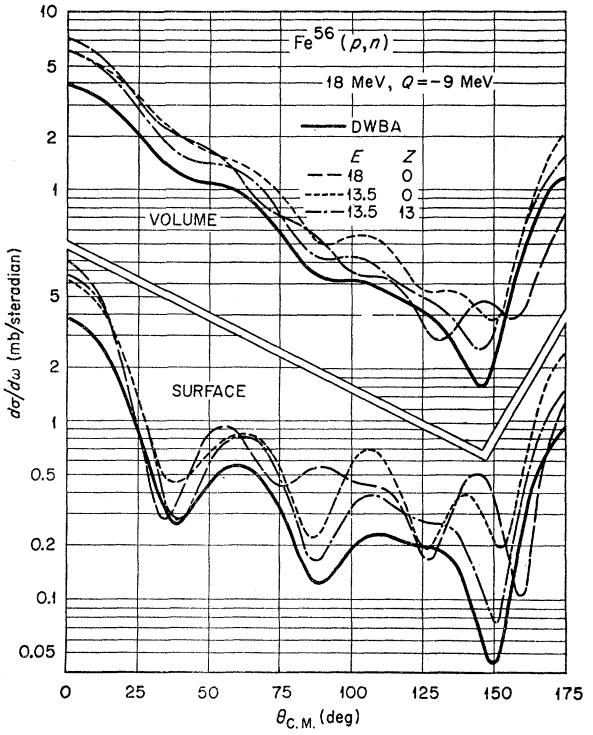

FIG. 7. Comparison of "exact" (DWBA) calculation with various forms of the adiabatic approximation. Both volume and surface potentials are shown, with $V_1=112$ MeV. Spin-orbit coupling is not included.

with those of the expression (4), which is exact in the adiabatic limit ($Q=0$) when the Coulomb interaction with the proton is neglected ($Z=0$). Nucleons of 18 MeV incident on ^{56}Fe were studied, with $V_1=112$ MeV and using both surface and volume forms for the i-spin interaction. With the volume term, the distorted-wave predictions differ from the exact by less than one part in 1000 everywhere. The surface interaction showed deviations of less than 1%. These results lead us to believe the distorted-wave approximation is also very accurate in the nonadiabatic case ($Q\neq0$) when the Coulomb field is acting ($Z\neq0$).

It has also been suggested[9] that the adiabatic expression (4) might give good results if instead of the full energy E_p, an effective energy $E_p+\frac{1}{2}Q$ were used (but still with $Z=0$). This energy is midway between the actual energies of the incident proton and emergent neutron. This was also tested for the same case, with $Q=-9$ MeV. As a natural extension, the calculation was repeated with $Z=13$, that is, half the actual charge of ^{56}Fe. The results are compared to the "exact" (distorted-wave) predictions in Fig. 7. While the adiabatic approximations reproduce the qualitative behavior of the differential cross sections, they always overestimate the magnitudes. This can be understood in the sense that the adiabatic approximation implies perfect overlap of the waves in the entrance and exit channels. Because of the more marked structure, the the deviations in shape of the angular distributions are greater in the surface case. It is interesting to note that the adiabatic approximation calculated with the average energy ($E_p=13.5$ MeV) and charge ($Z=13$) gives quite a good account of the angular distributions, but still over estimates the cross section by about 33% for the volume interaction and about 50% for the surface interaction. This discrepancy can be expected to increase as the nuclei become heavier, but the adiabatic approximation should give good results for light nuclei.[15]

Interaction Variations

In order to judge the significance of the comparisons with experiment discussed earlier it is necessary to know the sensitivity of the predictions to the assumptions made about the interaction. We have already compared extreme surface and volume forms; now Fig. 8 shows an intermediate case with equal amounts of surface and volume interaction. The predictions for this fall smoothly between the two extremes. On the basis of these results, it would be difficult to include much volume interaction without spoiling the fits to nuclei like Mn and Fe, but some other targets would not exclude appreciable volume contributions.

It is also of interest to relax the condition that the radius R_0 and diffuseness a_0 of the i-spin interaction have the same values as for the main optical potential. This is done for the surface interaction Eq. (12) in Fig. 9; in each case the solid curve represents the predictions

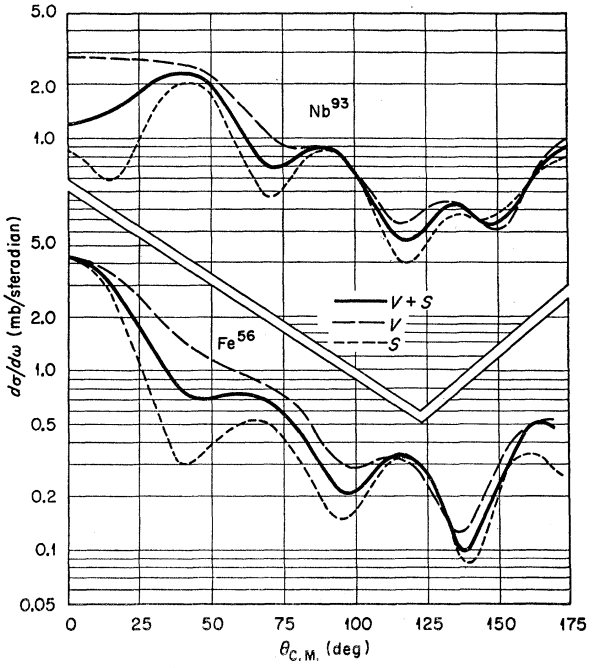

FIG. 8. Effect of using a symmetry potential with equal parts volume and surface ($V+S$) compared to extreme volume (V) and extreme surface (S). The energy is 18.5 MeV.

when the condition is satisfied. Increasing the radius R_0 sharpens the diffraction structure, and vice versa, whereas the principal effect of varying a_0 is to change the magnitude of the cross section. To some extent, the two effects are compensatory, as shown by the lowest curve, which results from a decrease in R_0 and increase in a_0. Since we may expect the interaction form factor resulting from a microscopic shell-model calculation to be somewhat of this type—broader and peaking inside the nucleus—it may require good experimental data to distinguish the two models.

Variations in R_0 and a_0 produce very little change in shape when the volume interaction is used, but the magnitude of the cross section increases roughly like $R_0^n(n\sim 2 \text{ to } 3)$ and $a_0^m(m\sim 2)$.

In general we should expect the i-spin potential to be complex, but we have little idea what shape to ascribe to the imaginary part. If its radial shape is the same as the real part, the angular distribution will be unaffected and only the magnitude of the cross section changed. A different radial shape could change the angular distribution also. However, a calculation was made with the volume real interaction plus an imaginary term of the surface shape, and another with the surface real interaction plus an imaginary term proportional to the derivative of the surface form. The ratio of imaginary to real strengths was assumed to be the same as the ratio W_D/V for the main potential. The effects on the (p,n) angular distributions were very small.

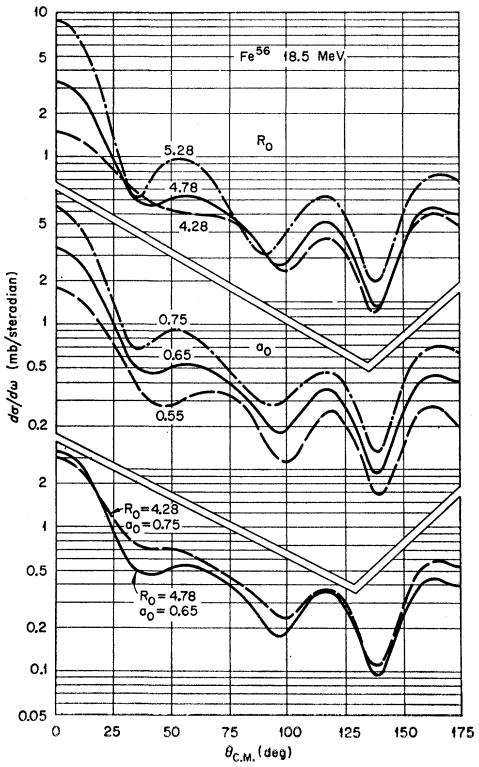

FIG. 9. Effects on the (p,n) predictions of varying the radius R_0 and diffuseness a_0 of the surface symmetry potential. The strength V_1 is fixed at 100 MeV.

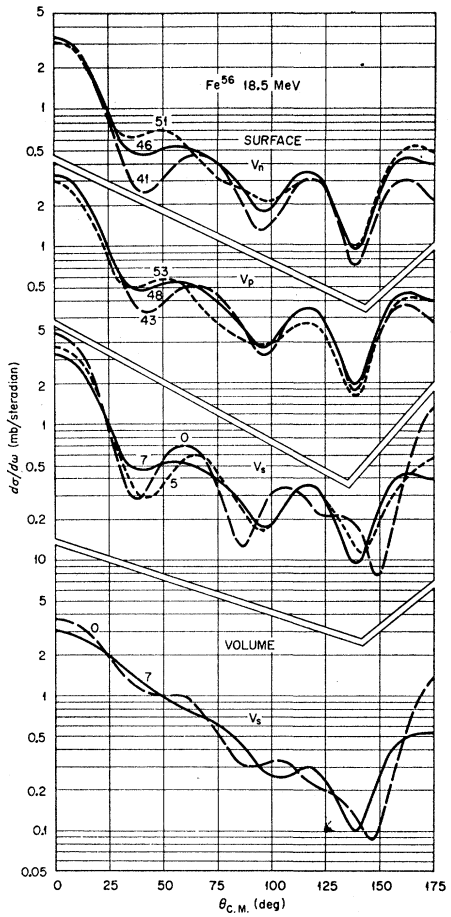

FIG. 10. Effects on the (p,n) predictions of varying the well depths of the real parts of the optical potential for neutrons (V_n) and proton (V_p), and the spin-orbit strength V_s of both.

Optical-Potential Variations

As has been repeatedly stressed above, there are uncertainties in the values of the parameters specifying the main optical potentials, so the consequences of small variations in these should also be studied. One such ambiguity is well known: Small variations in V and r_0 leave the elastic scattering unchanged provided Vr_0^n is kept constant, where n is approximately two.[20] The effect of this ambiguity on the (p,n) reaction was studied and it was found to be essentially unchanged also. This takes care of the problem of self-consistency in the calculations using the surface interaction; it is quite sufficient to adjust V instead of r_0 in the potentials generating the distorted waves. The elastic scattering is also insensitive to changes which keep the product $a'W_D$ constant, and again it was verified that this produces negligible change in the (p,n) predictions. Similarly, optical potentials with volume absorption

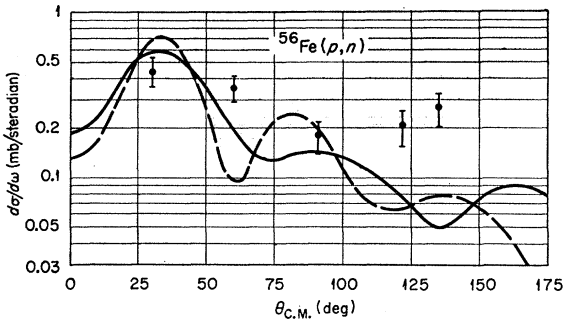

FIG. 11. Predicted angular distributions for "inelastic" (p,n) reaction at 17 MeV to the analog of the 845-keV 2^+ level in ^{56}Fe, using a nonspherical potential. The deformed volume interaction (solid curve) is normalized with $\beta=1.5$, the deformed surface interaction (broken curve) with $\beta=0.76$, and the V_1 values given in Table I are used.

which give closely the same elastic scattering also predict very similar (p,n) reactions.

Because of these results, it is only necessary to consider explicitly variations in V, W_D, and V_{so}. Changes in the absorption strength W_D for either proton or neutron only change the cross-section magnitude. In fact, we find that it is very closely proportional to $(W_{Dn}W_{Dp})^{-1}$. Otherwise the predictions for the volume symmetry potential are very stable against parameter variations. The surface potential is more sensitive, and the effects of changes in V and V_{so} are shown for ^{56}Fe in Fig. 10. (The effects of varying V_n for some other nuclei were shown in Figs. 1 to 3.) Perhaps the most surprising of these is the effect of reducing the spin-orbit strengths V_{so} from 7 to 5 MeV; the effect is qualitatively like that for a reduction in V. (However, an equivalent *increase* in V_s from 7 to 9 MeV was found to have very little effect.) The results of completely neglecting the spin-orbit coupling are also shown, both for the surface and the volume interactions; the effects are by no means negligible, although considerably less for the volume case. It was found for the heavier targets, $A\sim90$, that this sensitivity to small changes in the spin-orbit coupling has largely disappeared.

V. QUASI-INELASTIC TRANSITIONS

In addition to the "quasielastic" transitions we have been discussing which excite the isobaric analog of the target, we might expect scattering to states which are analogs of low excited states in the target nucleus, but still of essentially the same "configuration" as the ground state. These we may describe (conveniently, even if somewhat illogically) as "quasi-inelastic" transitions. We could expect to see these transitions when the corresponding true inelastic scattering from the target is strong, namely when the excited states are collective (rotational or vibrational) in nature.

Considerable success has been obtained in the description of inelastic scattering to these states through the use of another generalization of the optical-model potential, namely, allowing the potential to be nonspherical in either a static (rotational) or dynamic (vibrational) sense.[18,21] It seems reasonable to extend this concept to the isobaric-spin-dependent part of the potential. A quadrupole deformation in this potential would then excite the isobaric analog of the first 2^+ vibrational or rotational state of the target. An angular distribution was measured[9] for a (p,n) group from ^{56}Fe with $Q=-9.83$ MeV, which was interpreted as exciting the isobaric analog of the 845-keV 2^+ vibrational level in ^{56}Fe. The intensity of this group is roughly half that of the quasielastic group.

A way of treating deformed potentials has been discussed in detail elsewhere.[18,21] We allow the surface $r=R$ to be nonspherical, so that

$$R(\theta)=R_0[1+\sum_{lm}\alpha_{lm}Y_l^m(\theta,\phi)], \tag{18}$$

where the α_{lm} are the 2^l-pole deformation parameters. Using this in the volume form (10) of the symmetry potential and expanding to first order, we obtain

$$U_1^{(v)}(r,\theta,\phi)=V_1(\mathbf{t}\cdot\mathbf{T}_0/A) \times[f(x)-(R_0/a)(\alpha\cdot Y)df(x)/dx], \tag{19}$$

where

$$x=(r-R_0)/a,$$

and $(\alpha\cdot Y)$ is an abbreviation for $\sum_{lm}\alpha_{lm}Y_l^m(\theta,\phi)$. Similarly, the surface form (12) gives

$$U_1^{(s)}(r,\theta,\phi)=-V_1'(\mathbf{t}\cdot\mathbf{T}_0/A) \times[df/dx-(R_0/a)(\alpha\cdot Y)d^2f/dx^2]. \tag{20}$$

The second part of the expressions (19) and (20) allows quasi-inelastic scattering with transfer of angular momentum l.

Using this *ansatz* for the interaction, calculations were made for a quadrupole, $l=2$, transition in ^{56}Fe. Coulomb excitation and proton scattering indicate[18,21] a deformability $\beta\equiv\langle\sum_m|\alpha_{2m}|^2\rangle^{1/2}\simeq0.24$. With this strength, the surface form (20) gives an inelastic cross section which is an order of magnitude smaller than the $l=0$ quasielastic, while the volume form (19) predicts a cross section about 50 times smaller. The angular distributions are shown in Fig. 11 together with the experimental data; the curves are normalized with $\beta=1.5$ (volume) and 0.76 (surface). The results so far do not indicate even if this over-all approach is correct. Assuming it is, the significance of the large strengths, or deformabilities β, which are required is far from clear. One possible interpretation is that the nucleons outside closed shells contribute proportionately more to both the collective motion and the symmetry potential than the core nucleons. We would then expect the deformation of the symmetry potential to be greater than that of the main potential.

Another curious feature is that apparently while these

[21] B. Buck, Phys. Rev. **130**, 712 (1962).

"inelastic" groups are observed with even targets, they do not appear with odd-mass targets. The collective optical potential being used here would predict essentially the same quadrupole strengths for both types of nuclei, even though for odd targets it would be distributed amongst a narrow multiplet of levels.[22] It then becomes of great interest to obtain more precise data on these transitions to determine whether the potential model is adequate or whether a microscopic description of the transitions is necessary for their understanding.

As already remarked, when the target has a nonzero spin $I>\frac{1}{2}$, it is possible for other even multipoles $l>0$ to contribute to the "elastic" transition, up to the maximum $l=2I$. These could introduce odd-even differences in the (p,n) angular distributions. However, it is worth noting that the absence of such differences is not incompatible with the observation of strong "inelastic" transitions in vibrational nuclei. In such nuclei, the nonspherical part of the potential is only able (to lowest order) to excite a phonon and has no diagonal matrix elements.[23] However, this is no longer true for nuclei which exhibit a rotational spectrum, and a considerable part of the quadrupole strength for odd targets may appear in the "elastic" transitions.[23] For this reason it would be of considerable interest to examine a pair of nuclei such as ^{26}Mg and ^{25}Mg. The spin of ^{25}Mg is $\frac{5}{2}$, and in this case a fraction 5/14 of the quadrupole strength would contribute to the elastic scattering. A measure of the total quadrupole strength would be given by the cross section to the analog of the first 2^+ state in ^{26}Mg. Some data are available[9] for an "inelastic" group in ^{11}B(p,n), but the application of the simple potential model to such light nuclei is perhaps questionable.

VI. DISCUSSION

Two main features emerge from the present analysis. First is the surprising success of the simple potential model in reproducing the overall behavior of both the magnitude and the angular distribution of the experimental data. This has been achieved with essentially no free parameters. Although the strengths V_1 were adjusted in obtaining the fits shown in the figures, the variations in V_1 remain within a 20% range, and the values are close to those suggested by analysis of elastic scattering of protons.[5] Further, it is likely that some of these variations in V_1 arise from uncertainties in the experimental data or uncertainties over the optical-model parameters required to reproduce the elastic scattering of neutrons and protons.

The second feature is complementary, namely, there is a need for more complete and more precise measurements in order to establish the degree of adequacy of the model. This is particularly true of the "inelastic" scattering, for which there appears to be some anomalous behavior, as emphasized in the last section.

It would also be of interest to extend these measurements to higher energies, say 50 MeV. Calculations show that the characteristic differences between the predictions of the volume and surface interaction persist at these energies. Indeed, the additional diffraction structure introduced by the surface form may supply the explanation of the variations with target of the 0° cross sections at 30 and 50 MeV.[24] Results taken at these higher energies are also more likely to yield to a microscopic analysis in terms of shell-model wave functions, through the use of the impulse approximation for the nucleon-nucleon interaction. Since the (p,n) reaction singles out the i-spin-dependent part of this interaction, such studies would complement those on (p,p') inelastic scattering.

ACKNOWLEDGMENTS

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[24] C. J. Batty, G. H. Stafford, and R. S. Gilmore, Phys. Letters **6**, 292 (1963); and private communication.